

Identitati si inegalitati

1. Identitati:

Fie $a, b, c \in \mathbb{R}$ si $m, n \in \mathbb{N}$

- a) $(a \pm b)^2 = a^2 \pm 2ab + b^2$,
- b) $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$,
- c) $a^2 - b^2 = (a - b)(a + b)$,
- d) $a^3 \mp b^3 = (a \mp b)(a^2 \pm ab + b^2)$,
- e) $a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + \dots + ab^{n-2} + b^{n-1})$
- f) $a^{2m+1} + b^{2m+1} = (a + b)(a^{2m} - a^{2m-1}b + a^{2m-2}b^2 - \dots - ab^{2m-1} + b^{2m})$
- g) $a^{2m} - b^{2m} = (a^m - b^m)(a^m + b^m)$
- h) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.

2. Inegalitati:

- a) daca $0 < a \leq b$ atunci $a^p \leq b^p$ ($p \in \mathbb{R}^+$) si $\sqrt[q]{a} \leq \sqrt[q]{b}$ ($q \in \mathbb{R}$)
- b) $\left| |a| - |b| \right| \leq |a + b| \leq |a| + |b|$, $a, b \in \mathbb{R}$
- c) $|a + b + \dots + l| \leq |a| + |b| + \dots + |l| \quad \forall a, b, \dots, l \in \mathbb{R}$
- d) daca $a < b$ si $a, b, m, n > 0$ atunci $a < \frac{ma + nb}{m + n} < b$
- e) $a^2 + b^2 + c^2 \geq ab + bc + ca$, $\forall a, b, c \in \mathbb{R}$
- f) $a^3 + b^3 + c^3 \geq 3abc$, $\forall a, b, c \in \mathbb{R}$
- g) $\frac{a_1 + a_2 + \dots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \dots a_n}$, $a_1, a_2, \dots, a_n \in \mathbb{R}$
- h) $(1 + a)(1 + b) \dots (1 + t) > 1 + (a + b + \dots + t)$, $\forall a, b, \dots, t \geq 0$
- i) $1 - a^n < n(1 - a)$, $a > 0$,
- j) $n^{n+1} > (n + 1)^n$, $3 < n \in \mathbb{N}$,
- k) $\sqrt[n]{n^n} < n! < \left(\frac{n+1}{2}\right)^n$, $n \in \mathbb{N}^*$,
- l) $(a_1^2 + a_2^2 + \dots + a_n^2)(b_1^2 + b_2^2 + \dots + b_n^2) \geq (a_1 b_1 + a_2 b_2 + \dots + a_n b_n)^2$, $a_1, b_1, \dots, a_n, b_n \in \mathbb{R}$,
- m) daca $0 < a < 1$ si $x < y$ atunci $a^x > a^y$,
- n) daca $a > 0$ si $x < y$ atunci $a^x < a^y$,
- o) daca $0 < a < 1$ si $x < y$ atunci $\log_a x > \log_a y$,
- p) daca $a > 0$ si $x < y$ atunci $\log_a x < \log_a y$.